

Diaphragms shall be considered rigid when the maximum lateral deformation of the diaphragm is less than half the average interstory drift of the associated story. Diaphragms that are neither flexible nor rigid shall be classified as stiff. The interstory drift and diaphragm deformations shall be estimated using the seismic lateral forces from Section 5.3 or 5.4 of FEMA 302.

(2) Flexibility considerations. The in-plane deflection of the floor diaphragm shall be calculated for an in-plane distribution of lateral force consistent with the distribution of mass, as well as all in-plane lateral forces associated with offsets in the vertical seismic framing at that floor. The deformation of the diaphragm may be neglected in mathematical models of buildings with rigid diaphragms. Mathematical models of buildings with stiff diaphragms shall explicitly include diaphragm flexibility. Mathematical models of buildings with flexible diaphragms should explicitly account for the flexibility of the diaphragms. For buildings with flexible diaphragms at each floor level, the vertical lines of seismic framing may be designed independently, with seismic masses assigned on the basis of tributary area. Diaphragm flexibility results in: (1) an increase in the fundamental period of the building, (2) decoupling of the vibrational modes of the horizontal and vertical seismic framing, and (3) modification of the inertia force distribution in the plane of the diaphragm. There are numerous single-story buildings with flexible diaphragms. For example, precast concrete tilt-up buildings with timber-sheathed diaphragms are common throughout the United States. An equation for the fundamental period of a single-story building with a flexible

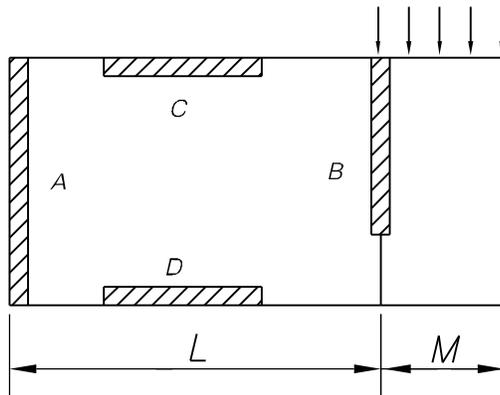
diaphragm is presented in the following equation:

$$T = (0.1\Delta_w + 0.078\Delta_d)^{0.5} \quad (7-5)$$

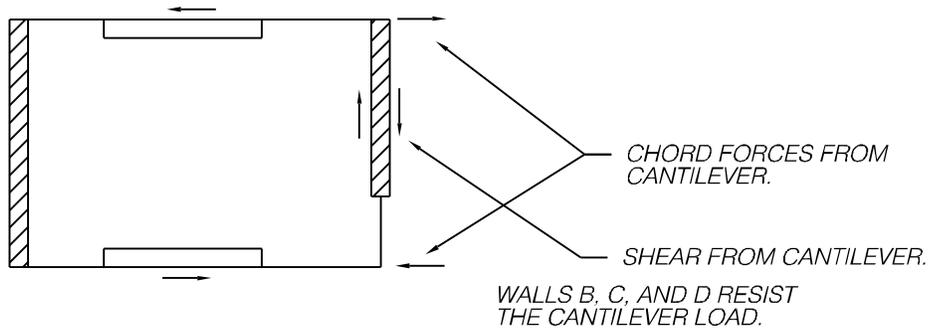
where  $\Delta_w$  and  $\Delta_d$  are in-plane wall and diaphragm displacements in inches, due to a lateral load, in the direction under consideration, equal to the weight of the building. For the displacements in mm, the calculated value of T shall be multiplied by 5. Wall displacements shall be estimated for each line of framing. For multiple-bay diaphragms, lateral load equal to the gravity weight tributary to the diaphragm bay under consideration shall be applied to each bay of the building to calculate a separate period for each diaphragm bay. The period so calculated that maximizes the equivalent base shear shall be used for design of all walls and diaphragms in the building.

(3) Rotation. In cases where there is a lack of symmetry either in the load or the reactions, the diaphragm will experience a rotation. Rotation is of concern because it can lead to vertical instability. This is illustrated in the following cases: the cantilever diaphragm, and the diaphragm supported on three sides.

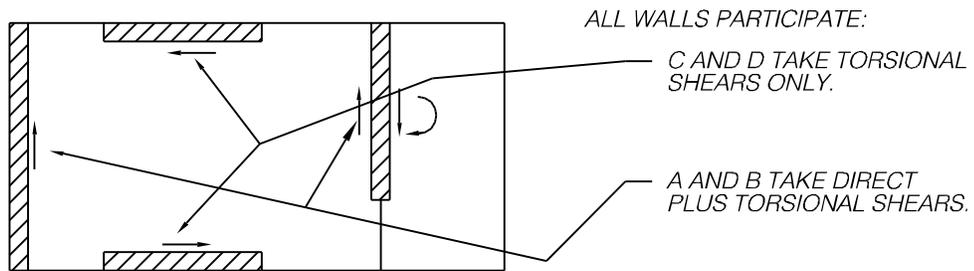
(a) Building with a cantilever diaphragm (an example is shown in Figure 7-48). The layout of the resisting walls is shown in Figure 7-48, part a. If the backspan is flexible relative to the walls (Figure 7-48, part b), the forces exerted on the backspan by the cantilever are resisted by walls B, C, and D, provided there are adequate collectors. If the backspan is relatively rigid (Figure 7-48, part c), the load from the cantilever is resisted by all four walls



A. THE WALLS



B. THE BACKSPAN - FLEXIBLE



C. THE BACKSPAN - RIGID

**Figure 7-48 Cantilever diaphragm.**

(A, B, C, and D); a rigidity analysis is needed in order to determine the forces in the walls.

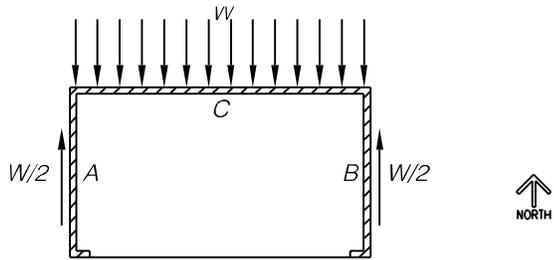
(b) Building with walls on three sides (an example is shown in Figure 7-49). For transverse (north-south) forces (Figure 7-49, part a), this is a simple case: because of symmetry of load and reactions, the end walls share the load equally. For longitudinal (east-west) forces (Figure 7-49, part b), there is an eccentricity between the resultant of the load and the centerline of the one east-west resisting wall, wall C. The analysis is simplified by treating the load as a combination of the load,  $W$ , acting directly on the wall, and the couple  $M = WD/2$  (Figure 7-49, part c). The direct force induces a direct shear,  $W$ , on the diaphragm and a reaction,  $W$ , in Wall C (Figure 7-49, part d); the moment,  $M$ , is resisted by walls A and B (Figure 7-49, part e), causing a counterclockwise rotation of the diaphragm. A particular concern with this type of building is the deflection on the corners at the open side. If this is excessive, it can lead to vertical instability in the southwest and southeast corners.

1. Flexible diaphragm. In an all-wood building, the concern about rotation is met by limitations on the size and proportions of the diaphragm. In buildings with walls of concrete or masonry, the greater weight causes greater concern for rotation, and there are special limitations on the span/width ratio of the diaphragms.

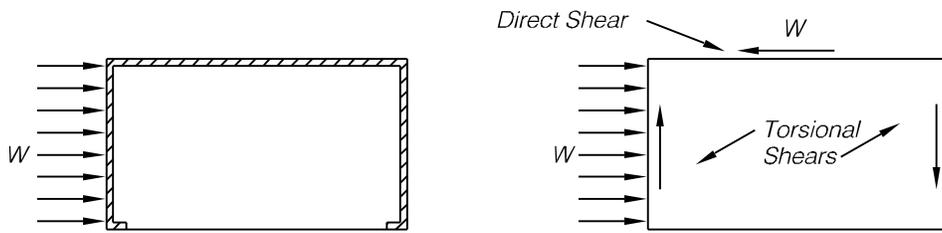
2. Rigid diaphragm. If the diaphragm is rigid, the design of the building will consider the effects of torsion. The concept of orthogonality does not apply.

(4) Torsion, in a general sense, occurs in a building whenever the location of the resultant of the lateral forces, i.e., the center of mass,  $cm$ , at and above a given level does not coincide with the center of rigidity,  $cr$ , of the vertical-resisting elements at that level. If the resisting elements have different deflections, the diaphragm will rotate. Torsion, in this general sense of rotation, occurs regardless of the stiffness properties of the diaphragms and the walls or frames. For purposes of design, however, the procedure for dealing with torsion does depend on these stiffness properties.

(a) Flexible diaphragms. Flexible diaphragms such as wooden diaphragms can rotate, but cannot develop torsional shears. For example, a single-span diaphragm with a relatively stiff shear wall at one end and a more flexible frame at the other end will rotate because the two resisting elements have different deflections. Flexible diaphragms, however, are considered incapable of inducing forces in the walls or frames that are perpendicular to the direction of the design forces; i.e., flexible diaphragms are said to be incapable of taking torsional moments. All of the lateral load is taken by the walls that are parallel to the lateral forces; none is taken by the other walls. (The building with walls on three sides is a special case and entails special limitations, as discussed above.) Lateral loads are usually distributed to the resisting walls by using the continuous beam analogy. There is no rigidity analysis, no calculation of the  $cm$  and the  $cr$ . If there are uncertainties about the locations of the loads and the rigidities of the structural elements, the design can be adjusted to bracket the range of possibilities.

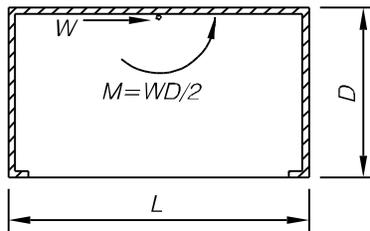


A. N-S Forces

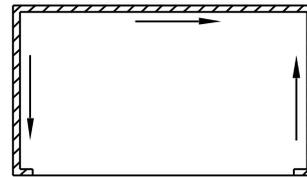


B. E-W Forces

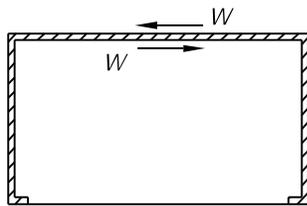
Free body diag. of diaphragm



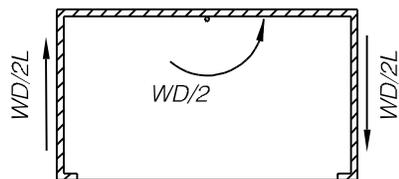
C. Equivalent E-W load.



Forces on walls



D. Direct component of E-W load



E. Torsional component of E-W load

**Figure7-49 Building with walls on three sides.**

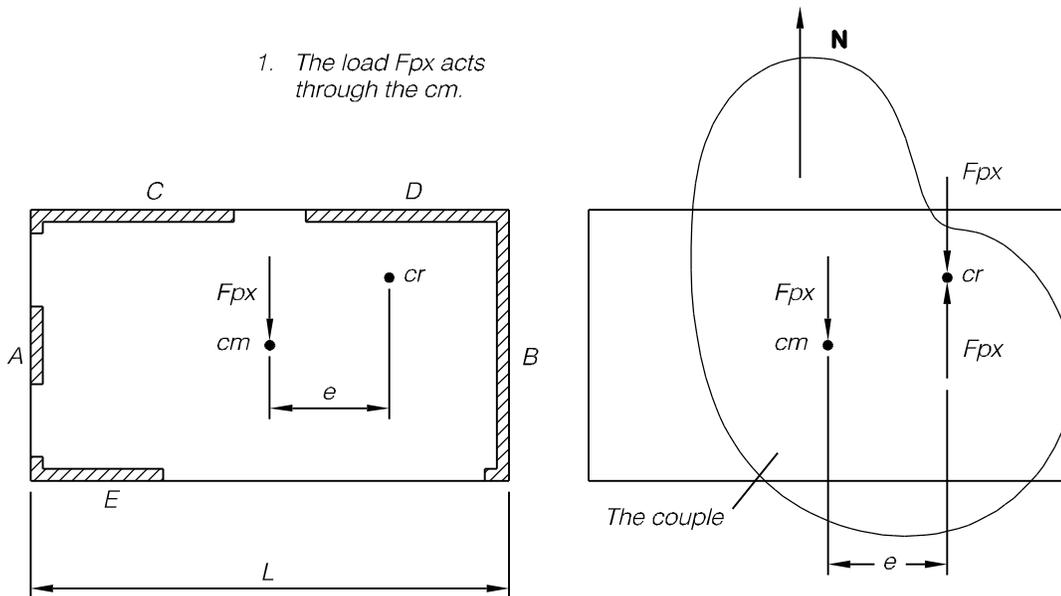
(b) Rigid diaphragms. When rigid diaphragms rotate, they develop shears in all of the vertical-resisting elements. In the example (Figure 7-50) there is an eccentricity in both directions, and all five walls develop resisting forces via the diaphragm.

(c) Deformational compatibility. When a diaphragm rotates, whether it is rigid or flexible, it causes displacements in all elements attached to it. For example, the top of a column will be displaced with respect to the bottom. Such displacements must be recognized and addressed.

(d) Analysis for torsion. The method of determining torsional forces is indicated in Figure 7-50. The diaphragm load,  $F_{px}$ , which acts through the  $cm$ , is replaced by an equivalent set of new forces. By adding equal and opposite forces at the  $cr$ , the diaphragm load can now be described as a combination of a force component,  $F_{px}$  (which acts through the  $cr$ ) and a moment component (which is formed by the couple of the two remaining forces  $F_{px}$  separated by the eccentricity  $e$ ). The moment, called the torsional moment,  $M_T$ , is equal to  $F_{px}$  times  $e$ . The torsional moment is often called the “calculated” torsion, because it is based on a calculated eccentricity; also this name distinguishes it from the “accidental” torsion, which is described below. In the modified loading, the force  $F_{px}$  acts through the  $cr$  instead of  $cm$ ; therefore, it causes no rotation and it is distributed to the walls, which are parallel to  $F_{px}$  in proportion to their relative rigidities. The torsional moment is resolved into a set of equivalent wall forces by a procedure similar to that used for finding forces on bolts in an eccentrically loaded group of bolts. The formula is analogous to the torsion formula  $t = Tc/J$ . The torsional shear forces can

thus be expressed by the formula  $F_i = M_T kd / \Sigma kd^2$ , where  $k$  is the stiffness of a vertical-resisting element,  $d$  is the distance of the element from the center of rigidity, and  $\Sigma kd^2$  represents the polar moment of inertia. For the wall forces, the direct components due to  $F_{px}$  at the  $cr$  are combined with the torsional components due to  $M_T$ . In the example shown on Figure 7-50, the torsional moment is counterclockwise, and the diaphragm rotation will be counterclockwise around the  $cr$ . The direct component of the load is shared by walls A and B, while the torsional component of the load is resisted by walls A, B, D, C, and E. Where the direct and torsional components of wall force are the same direction, as in wall A, the torsional component adds to the direct component; where the torsional component is opposite to the direct component, as in wall B, the torsional component subtracts from the direct. Walls C, D, and E carry only torsional components; in fact, their design will most likely be governed by direct forces in the east-west direction.

(e) Accidental torsion. Accidental torsion is intended to account for uncertainties in the calculation of the locations of the  $cm$  and the  $cr$ . The accidental torsional moment,  $M_A$ , is obtained using an eccentricity,  $e_{acc}$ , equal to 5 percent of the building dimension perpendicular to the direction of the lateral forces; in other words,  $M_A = F_{px} \times e_{acc}$ . For the example of Figure 7-50, the accidental torsion for forces in the north-south direction is  $M_T = F_{px} \times 0.05L$ . In hand calculations,  $M_A$  is treated like  $M_T$ , except that absolute values of the resulting forces are



2. Add an equal and opposite force,  $F_{px}$  at the  $cr$ .

3. Use the equivalent load:

- a.  $F_{px}$  acting through the  $cr$
- b. The moment formed by the two remaining  $F_{px}$ 's. The torsional moment is

$$M_T = F_{px} \times e$$

**Figure7-50 Calculated torsion.**

used so that the accidental torsion increases the total design force for all walls. In computer calculations, the accidental torsion may be handled by running one analysis, using for eccentricity the calculated eccentricity plus the accidental eccentricity, then running a second analysis, using the calculated minus the accidental eccentricity, and finally, selecting the larger forces from the two cases.

(f) Dynamic amplifications of torsion. Section 5.3.5.3 of FEMA 302 specifies dynamic amplifications for Type 1 torsional irregularities in Seismic Design Category C, D, E, and F structures analyzed by the ELF procedure.

(5) Flexibility limitations. The deflecting diaphragm imposes out-of-plane distortions on the walls that are perpendicular to the direction of lateral force. These distortions are controlled by proper attention to the flexibility of the diaphragm. A diaphragm will be designed to provide such stiffness that walls and other vertical elements laterally supported by the diaphragm can safely sustain the stresses induced by the response of the diaphragm to seismic motion.

(a) Empirical rules. Direct design is not feasible because of the difficulty of making reliable calculations of the diaphragm deflections; instead, diaphragms are usually proportioned by empirical rules. The design requirement is considered to be met if the diaphragm conforms to the span and span/depth limitations of Table 7-24. These limitations are intended as a guide for ordinary buildings. Buildings with unusual features should be treated with caution. The limits of Table 7-24 may be exceeded, but only when justified by a reliable

evaluation of the strength and stiffness characteristics of the diaphragm. If the diaphragm is providing out-of-plane lateral support to the top of a relatively short or stiff concrete or masonry wall, it should be noted that wall will experience the diaphragm deflections plus the in-plane deflection of the vertical lateral-load-resisting system. For use of Table 7-24, the flexibility category in the first column of the table can be determined with little or no calculation: concrete diaphragms are rigid; bare metal deck diaphragms can be stiff or flexible; plywood diaphragms can be considered to be rigid when used in light wood framing, but should be considered to be stiff or flexible with other framing systems; special diaphragms of diagonal wood sheathing are flexible; and conventional diaphragms of diagonal wood sheathing and diaphragms of straight wood sheathing are very flexible (very flexible diaphragms are seldom used in new construction because of their small capacities).

(b) Diaphragm deflections. When a deflection calculation is needed, the following procedure will be used.

1. Deflection criterion. The total deflection of the diaphragm under the prescribed static forces will be used as the criterion for the adequacy of the stiffness of a diaphragm. The limitation on the allowable amount relative to out-of-plane deflection (drift) of the walls, between the level of the diaphragm and the floor below, is equal to the deflection of the orthogonal walls at the ends of the diaphragm, plus the deflection of the diaphragm, as

		Diaphragm Span / Diaphragm Depth Limitations	
Flexibility Category	Allowable Span of Diaphragm, ft.*	Concrete or Masonry Walls	Other Walls
Flexible	100	2:1	2½ :1
Stiff	200	2½ :1	3½ :1
Rigid	350	3½ :1	4:1

\*1 foot = 0.3m

**Table 7-24: Span and Depth Limitations on Diaphragms**

computed in the following paragraphs.

2. Deflection calculations. The total computed deflection of diaphragms ( $\Delta_d$ ) under the prescribed static seismic forces consists of the sum of two components: the first component is the flexural deflection ( $\Delta_f$ ); the second component is the shearing deflection ( $\Delta_w$ ). When most beams are designed, the flexural component is usually all that is calculated, but for diaphragms, which are like deep beams, the shearing component must be added to the flexural component.

i. Flexural component. This is calculated in the same way as for any beam. For example, for a simple beam with uniform load, the flexural component is obtained from the familiar formula  $\Delta_f = 5wL^4/384EI$ . The only question is the value of the moment of inertia,  $I$ . For diaphragms whose webs have uniform properties in both directions (concrete or a flat steel plate), the moment of inertia is simply that of the diaphragm cross-section. For diaphragms of fluted steel deck, or diaphragms of wood, whose stiffness is influenced by nail slip and chord-joint slip, the flexural resistance of the diaphragm web is generally negligible, and the moment of inertia is based on the properties of the diaphragm chords. For a diaphragm of depth  $D$  with chord members each having area  $A$ , the moment of inertia,  $I$ , equals  $2A(D/2)^2$ , or  $AD^2/2$ .

ii. Shearing Component. The shearing component of deflection can be derived from the following equation:

$$\Delta_w = \frac{q_{ave} L_1 F}{10^6} \quad (7-6)$$

where:

$\Delta_w$  = web component of diaphragm deflection, in. (mm).

$q_{ave}$  = average shear in diaphragm, lbs. /ft. (N/m).

$L_1$  = distance from adjacent vertical resisting element (i.e., such as a shear wall) and the point to which the deflection is to be determined, ft. (m).

$F$  = flexibility factor, micro inches per foot of span stressed with a shear of one pound per foot (micro millimeters per meter of span stressed with a shear of one Newton per meter of span).

Values of the flexibility factor,  $F$ , and the allowable shear per foot,  $q_D$ , for steel decking are given in manufacturers' catalogs, as well as the Diaphragm Design Manual of the Steel Deck Institute (SDI). Deflection calculations for concrete diaphragms are seldom required, but the deflection can be calculated by the conventional beam theory. For example, for a diaphragm with a single span of length,  $L$ , with a total load,  $W$ , that is uniformly distributed, the maximum shearing deflection is:

$$\Delta_w = \frac{aWL}{8A_wG} \quad (7-7)$$

where:

$a$  = a form factor ( $L/D$  for prismatic webs)

$A_w$  = area of the web

$G$  = the shear modulus

noting that:

$R$ , the end reaction, equals  $W/2$  and  $q_{ave} = R/2D = W/4D$ ,  $L = 2L_1$ , and  $A_w = Dt$

Where  $t$  is the thickness of the web, and  $D$  is the depth of the diaphragm, the formula for shearing deflection can also be expressed as:

$$\Delta_w = \frac{q_{ave} L_1 a}{tG} \quad (7-8)$$

As noted above, this is only applicable to webs of uniform properties. For a concrete slab with  $\alpha = 1.5$ ,  $G = 0.4 E$ , and  $E = 33w^{1.5} \sqrt{f'_c}$ , the formula in English units becomes:

$$\Delta_w = \frac{q_{ave} L_1}{8.8tw^{1.5} \sqrt{f'_c}} \quad (7-9)$$

where:

$t$  = thickness of the slab, in.

$w$  = unit weight of the concrete, lbs. /cu. ft.

Recent editions of the SDI Diaphragm Design Manual provide the following alternative equation for the deflection of steel deck diaphragms:

$$\Delta_w = \frac{wL^2}{8DG'} \quad (7-10)$$

where:

$w$  = uniform lateral shear load on the diaphragm, K/ft. (N/m).

$L$  = diaphragm span, ft. (m).

$D$  = depth of diaphragm, ft. (m).

$G'$  = effective shear modulus calculated from tabulated values based on profile and thickness of deck and type and spacing of connectors.

The effective shear modulus,  $G'$ , is related to the flexibility factor,  $F$ , as follows:

$$G' = 10^3 / F \quad (7-11)$$

c. *Design of Diaphragms.* A deep-beam analogy is used in the design. Diaphragms are envisioned as deep beams with the web (decking or sheathing) resisting shear and the flanges (spandrel beams or other members) at the edges resisting the bending moment.

(1) Unit shears. Diaphragm unit shears are obtained by dividing the diaphragm shear by the length or area of the web, and are expressed in pounds per foot (N/m) (for wood and metal decks) or pounds per square inch (MPa) (for concrete). These unit shears are checked against allowable values for the material. Webs of precast concrete units or metal-deck units will require details for joining the units to each other and to their supports so as to distribute shear forces.