APPENDIX A

DEVELOPMENT OF A HOPKINS STATISTIC FOR UXO CALCULATOR

Development of a Hopkins Statistic for UXO Calculator
(Draft Report)

Introduction. The U.S. Army Engineering and Support Center, Huntsville (HNC) is responsible for the ordnance cleanup of formerly used defense sites. HNC has developed a software package for the statistical evaluation of sites undergoing sampling. The tool is known as UXO Calculator. HNC needs a sector spatial statistic that can be integrated with this tool. This report provides this statistic and two examples of its use.

The organization of this report is as follows. The first section provides an explanation of the spatial statistic. This includes the definition of all variables, units, etc., as well as the relevant mathematical expressions.

The second section contains several examples applying the spatial statistic to sampling scenarios. These examples are used to display the ability of the spatial statistic to indicate homogeneous vs. non-homogeneous sectors.

Discussion of the Spatial Statistic

The spatial statistic chosen is derived from the Hopkins-like statistic used in SiteStats. The main difference here is the lack of sampling grids in UXO Calculator. The spatial statistic algorithm is invoked when UXO Calculator has concluded that sufficient sampling has been conducted to satisfy a conclusion associated with UXO density or count.

Variable Definitions:

N = total number of UXO found during investigation,
U = average Euclidean distance (feet) from centroid of sampled subareas to nearest UXO location,
W = average Euclidean distance (feet) from each UXO location to its nearest UXO neighbor.

Sector Hypotheses:

A. Null Hypothesis: Sector is homogeneous w.r.t. UXO density.
B. Alternate Hypothesis: Sector is non-homogeneous w.r.t. UXO density.

Spatial Statistic Algorithm:

1. If N = 0 or N = 1, do not reject null hypothesis. Operationally, conclude the sector is homogeneous.
2. Otherwise, calculate $H = \frac{U}{U+W}$.

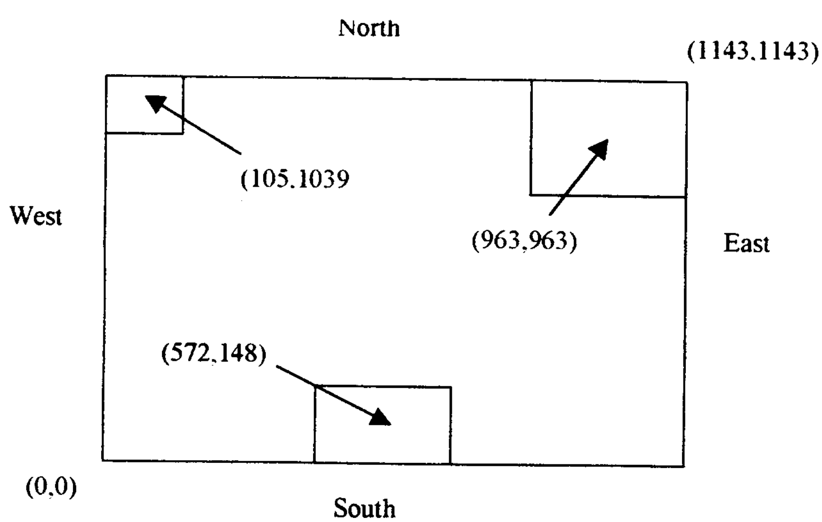
3. If H > BETAINV(0.95, N, N), reject null hypothesis. Operationally, conclude the sector is non-homogeneous. Otherwise, do not reject null hypothesis, and operationally conclude the sector is homogeneous w.r.t. UXO density.
4. Stop.

BETAINV is a standard spreadsheet calculation (e.g., Microsoft Excel) for the inverse beta distribution. Below is a sample table of values for various significance levels and number of UXO found. As intuition would indicate, it becomes easier to conclude non-homogeneity as the significance level decreases and the number of UXO found increases.

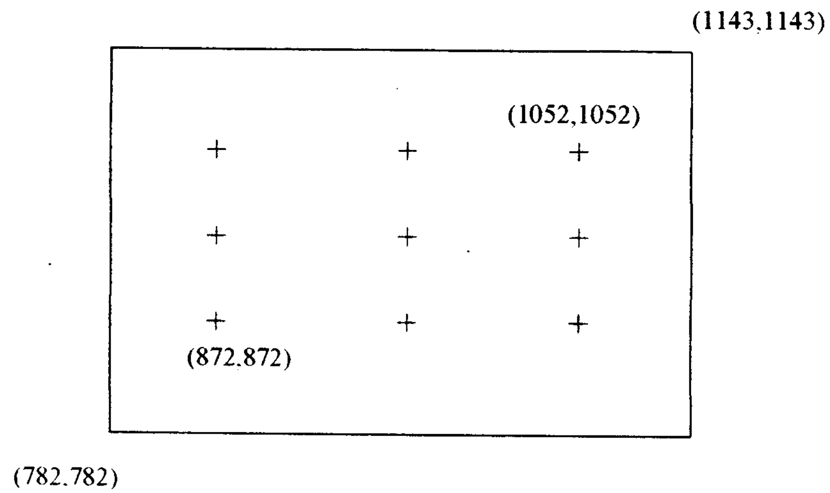
Significance Level	N = 2	N = 4	N = 6	N = 9
0.99	0.941	0.858	0.806	0.758
0.95	0.865	0.775	0.729	0.689
0.9	0.904	0.721	0.682	0.650

Example 1

Consider the following scenario:
A 30-acre square sector is being investigated. Three sampled subareas have been investigated. The NE subarea is square, 3 acres in size, and 9 UXO items were found uniformly distributed in the subarea. The NW subarea is square, 1 acre in size, with no UXO items discovered. The south-central subarea is square, 2 acres in area, also with no UXO items found. The locations of the centroids of each subarea and the UXO items discovered was determined using GPS. The SW corner of the sector was also identified using GPS, and serves as the sector origin. The first "map" below shows the sector of interest, with locations of the centroids of the three sampled subareas. The second "map" shows an expanded view of the NE subarea and the locations of the discovered UXO items, shown as "+" signs.



Map 1. Sector of Interest



Map 2. NE Subarea Expanded

First, the average distance from subarea centroids to nearest UXO item is calculated as:

$$U = \frac{\sqrt{(872-572)^2+(872-148)^2}+\sqrt{(105-872)^2+(1039-1052)^2}+\sqrt{(962-962)^2+(962-962)^2}}{3}$$

$$= 516.93$$

Next, the average distance from each UXO to its nearest UXO neighbor is calculated as:

$$W = \frac{9(90)}{9} = 90$$

The spatial statistic is calculated as:

$$H = \frac{516.93}{516.93+90} = 0.85 > 0.69 = BETAINV(0.95,9,9)$$

Therefore, there is a strong indication that this sector is non-homogeneous w.r.t. UXO density.

Example 2

Next, consider a slightly changed example, where only two UXO items are discovered. Suppose that both UXO are in the NE sector and are the most SW and NE UXO items in Map 2. Then, the calculations are given as:

$$U = \frac{\sqrt{(872-572)^2+(872-148)^2}+\sqrt{(105-872)^2+(1039-872)^2}+\sqrt{(962-872)^2+(962-872)^2}}{3}$$

$$= 565.3$$

$$W = \frac{2\sqrt{(1052-872)^2+(1052-872)^2}}{2} = 254.6$$

$$H = \frac{565.3}{565.3 + 254.6} = 0.69 < 0.86 = BETAINV(0.95,2,2)$$

Therefore, operationally, one would conclude that this sector is non-homogeneous w.r.t. UXO density.

Example 3

Finally, consider the following variation on the example. Suppose, again, that 2 UXO items were discovered. However, this time suppose that the first item found was the most SW UXO in the NE subarea of Map 2, and that the second UXO item was discovered at coordinates (572,100) of the south-central subarea of Map 1. Then, the resulting calculations are:

$$U = \frac{\sqrt{(572-572)^2 + (100-148)^2} + \sqrt{(105-872)^2 + (1039-872)^2} + \sqrt{(962-872)^2 + (962-872)^2}}{3}$$
$$= 342.3$$

$$W = \frac{2\sqrt{(572-872)^2 + (100-872)^2}}{2} = 828.2$$

$$H = \frac{342.3}{342.3 + 828.2} = 0.29 < 0.86 = BETAINV(0.95,2,2)$$

Again, operationally, one would conclude that this sector is non-homogeneous w.r.t. UXO density. Furthermore, as expected, this sector would indicate even weaker non-homogeneity than the previous sector considered.